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COMMENTS ON THE PAPER

VERTICAL VIBRATION OF A RIGID CIRCULAR BODY AND HARMONIC ROCKING OF A RIGID RECTANGULAR BODY ON AN ELASTIC STRATUM [1]

THE mixed boundary value problem of the vertical vibration of a rigid circular body on an elastic stratum has been formulated in Awojobi's paper, in terms of the dual integral equations (28). By making certain approximations these equations were reduced to a new pair of dual integral equations which correspond to the problem of vertical vibrations of the body on an elastic half space. Thus the author was able to conclude that "a semi-infinite elastic medium vibrating at a lower frequency factor η_{2e} and of the same elastic constants is approximately equivalent to the above stratum vibrating at the frequency factor η_2. The approximation is valid for all values of $\eta_2 > 1/\tilde{h}$ and improves with increasing $\tilde{h}$". This result implies that the steady-state vertical response of a rigid circular body with mass on an elastic stratum would always be finite as the response of the body on an elastic half-space is bounded. This, however, is in contradiction with the result obtained by Bycroft [2], Warburton and more recently by Kashio [3] that there will be no geometrical damping in the elastic layer for values of the frequency factor η_2 less than a given threshold value, η_t, which depends on the ratio $\tilde{h}$ and Poisson's ratio, ν. Thus the maximum response of a circular rigid body with mass will be infinite if its resonant frequency falls in the range $\eta_2 \leq \eta_t$. For example, Bycroft [2] and Warburton have shown that $\eta_t = \pi/\sqrt{2}\tilde{h}$ for $\nu = 0$. Bycroft [2] has shown also that the real part of the compliance function relating the vertical displacement of the massless circular rigid body to the force required to produce this displacement goes to infinity, indicating resonance, if $\eta_2 = [(2n-1)\pi]/2\gamma\tilde{h}$, $n = 1, 2, 3, \ldots$, where $\gamma^2 = (1-2\nu)/[2(1-\nu)]$. The response of the half-space model of the elastic stratum suggested by the author, however, is always finite.

The differences in behavior noted above are consequences of the approximations made by the author in transforming equations (28) into equations (30). The major approximation is introduced when the term $\alpha_2 \coth(\alpha_2\tilde{h})$ in the integrand of the first equation (28) is replaced by $\sqrt{(\eta^2-\eta_{2e}^2)}$. Awojobi showed in Table 1 that the resulting exact and approximate expressions are in good agreement for values of η greater than η_2. For $\eta < \eta_2$, however, the substitution

$$\alpha_2 \coth(\alpha_2\tilde{h}) \simeq \sqrt{(\eta^2-\eta_{2e}^2)} \tag{29a}$$

is not justified as α_2 becomes an imaginary number and the hyperbolic cotangent becomes a trigonometric cotangent. The author argues that for low frequency factor vibrations the substitution (29a) is permissible as the main contributions to the exact integral come from values of $\eta > \eta_2$. This is simply not true. For instance, Bycroft [2] has shown that the integrand in the first equation (28) has a *non-integrable singularity at the origin* if $\eta_2 = \pi/2\gamma\tilde{h}$.

By replacing the integrand in the first of equations (28) for that of the corresponding equation (30), the author has neglected to consider the poles characteristic of the stratum problem. It may be noted also that whereas the integrand in the first of equations (30) has branch points at $\eta = \eta_{1e}$ and at $\eta = \eta_{2e}$, the integrand in the corresponding equation (28) is free of branch points.

A suitable solution to the problem under discussion may be found by reducing the pair of dual integral equations (28) to a Fredholm integral equation of the second kind which may then be solved numerically. This method was used by Kashio [3] to solve the problem of a rigid disk resting on a stratum underlain by an elastic half space. The problem of the stratum on a fixed base was solved as a particular case.

Comments similar to the ones in this discussion would apply to the problems of the harmonic rocking of a rigid rectangular body and the torsional vibration of a rigid circular body on an elastic stratum.

REFERENCES

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